J. Fluid Mech. (1969), *vol.* 36, *part* 2, *pp.* 367–383
Printed in Great Britain

A computational method for the flow through non-uniform gauzes: the general two-dimensional case

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(Received 9 May 1968)

A computational method is presented for the analysis of two-dimensional flow through a non-uniform gauze. The method, based upon the linearized theory due to Elder (1959), permits solutions for most practical cases to be obtained using relatively simple numerical techniques. Comparison with experimental data shows that the computed solutions are satisfactory provided the restrictions inherent in the linearized theory are observed.

Introduction

Experimentally, the generation of a particular velocity distribution in a ducted flow is a problem of some interest. Several methods are available, which use the pressure loss associated with bluff bodies or wire grids, for example, to redistribute the upstream flow and thereby produce the desired velocity distribution downstream. When consideration is given to a particular method, ease of design and the magnitude of the disturbance suffered by the flow are the relevant factors (Livesey, Turner & Glasspoole 1966). These requirements, and the advantages offered by a semi-theoretical approach, will generally eliminate the cruder empirical methods. Thus attention is concentrated upon techniques which employ grids of parallel circular rods or wire gauzes with non-uniform physical properties. Conveniently, the term 'gauze' may also be applied to an array of parallel rods.

The influence of a non-uniform gauze on a two-dimensional flow has been discussed several times in the literature. The earliest analysis due to Owen & Ziekiewicz (1957), results in an expression for the spacing of a circular rod array, graded to produce a linear shear distribution. A constant velocity distribution is assumed upstream of the grid. Subsequently, Livesey & Turner (1964) extended the method to cover the generation of a symmetrical linear shear profile; for many practical situations this symmetry is desirable.

Probably the most complete work on the subject is that due to Elder (1959) who derived a linear relationship between the non-uniform gauze properties and the velocity distributions in the upstream and downstream flows. The result due to Owen & Zienkiewicz (1957) is shown to be a special case of this linearized theory. Close agreement with experiment was obtained for a plane inclined gauze and a parabolic gauze shape. The inverse problem, to produce a specified downstream distribution of velocity using a shaped gauze with uniform properties, was not checked experimentally. Although the theory given by Elder is perfectly general and covers all aspects of the problem, the analytical complexity is considerable even for carefully chosen problems. This is undoubtedly the reason why the method has not found a wider acceptance in experimental work.

McCarthy (1964) considered the case of moderately sheared three-dimensional flow past a plane normal gauze of arbitrary resistance distribution in a duct of arbitrary (although constant) cross section. The analysis, in certain respects, represents an extension of that due to Elder, since no restriction is placed on the resistance variation across the grid or on velocity variations across the duct. A relatively simple numerical calculation procedure is presented and the method is verified experimentally.

Examination of the literature therefore indicates that the analysis due to Elder is the most satisfactory. The theory derived by McCarthy, although more suitable for the analysis of highly sheared flows, is not applicable to gauzes of arbitrary shape or to non-uniform upstream flow distributions.

It was felt desirable that some attempt should be made to produce a more easily used method of analysis and yet retain the generality offered by Elder's method. Accordingly, numerical methods of solution have been developed which are suitable for digital computation. These use the basic analytical relationships derived by Elder but solve the problem by means of simple iterative techniques. Hence, any two-dimensional flow through a non-uniform gauze becomes amenable to analysis. Several problems are analyzed using the numerical methods and the results are found to be in satisfactory agreement with experiment.

Basic analysis

Initially, it appears worthwhile to re-state the main steps in the analysis of the general two-dimensional problem, as originally propounded by Elder (1959). Certain minor changes to the notation are made in order to clarify particular features.

In the ideal model considered, the gauze is replaced by a surface across which a discontinuity in the pressure and velocity distributions of the flow will occur. The fluid must satisfy continuity and also obey certain conditions at the gauze which may be specified in terms of geometrical and empirical parameters.

Gauze parameters

(i) *Loss coefficient*

The gauze loss coefficient K is defined by the expression

$$\Delta p = K(\tfrac{1}{2}\rho U_N^2),$$

where U_N is the local velocity component (in the upstream flow) normal to the gauze surface, ρ the fluid density and Δp the static pressure loss across the gauze. The loss coefficient depends on the gauze geometry and the interstitial velocity of the flow.

Defining the fractional open area β as the ratio of the free to the total gauze area

$$\beta = \{1-(d/l)\}^2 \quad \text{for a square mesh gauze,}$$

or

$$\beta = \{1-(d/l)\} \quad \text{for a grid of parallel rods,}$$

where d represents the wire diameter, and l is the wire spacing. Elder (1959) assumed the approximate expression

$$K = \{(1-\beta)/\beta\}^2. \tag{1a}$$

McCarthy (1964), however, has suggested the relationship

$$K = \{C_1(1-\beta)\}/\beta^2, \tag{1b}$$

where the constant C_1 is found to depend on the Reynolds number of the flow through the gauze, based upon the interstitial velocity $\overline{U}d/\beta\nu$. A value of $C_1 = 0{\cdot}78$ was chosen as the most satisfactory based on the experimental evidence. This expression (1a) is similar to that adopted by Owen & Zienkiewicz (1957).

A third relationship, obtained by Annand (1953), is

$$K = \{C_2(1-\beta^2)\}/\beta^2, \tag{1c}$$

where the constant C_2 is close to $0{\cdot}71$ for a Reynolds number $\overline{U}d/\nu$ of 100.

These empirical relationships are conflicting and lead to significantly different values of the loss coefficient. For the present purpose, however, it is assumed that the value of K can be determined accurately.

(ii) *Lift coefficient*

The gauze will, in general, experience a lift force normal to the flow direction so that there is a change in the tangential velocity component across the gauze. Thus the gauze lift coefficient may be defined as

$$B = (V_{s1}-V_{s2})/V_{s1}$$

where V_s denotes the velocity component tangential to the gauze and suffices 1, 2 represent upstream and downstream conditions respectively. For the present purpose, we may take B as a constant, in the range 0–1, dependent only on the physical gauze properties but independent of the inclination of the flow to the gauze.

It will be found that there are alternative empirical relationships for the lift coefficient in the literature. Elder (1959) showed

$$B = d/l. \tag{2a}$$

McCarthy (1964) defined a refraction coefficient α such that $\alpha = V_{s2}/V_{s1}$. Based on the results of Spangenberg, the empirical relationship found is

$$\alpha = 1{\cdot}1/\surd(1+K)$$

and hence

$$B = 1-1{\cdot}1/\surd(1+K). \tag{2b}$$

Expression (2b) is identical to that used by Owen & Zienkiewicz (1957) and appears to be rather more firmly based on experimental data than the alternative relationship.

Assuming the value of the loss coefficient is accurate, equations (2a) and (2b) can lead to very different values for the lift coefficient. Such discrepancies are important in the problems to be analyzed subsequently and thus particular care will be needed when evaluating this parameter.

The equation of motion for the flow through the gauze

Ideal two-dimensional flow along a parallel-walled channel is considered. The gauze occupies the range $0 < y < L$ close to the plane $x = 0$ and the streamlines in the flow far from the gauze are assumed parallel to the walls. The model is shown schematically in figure 1.

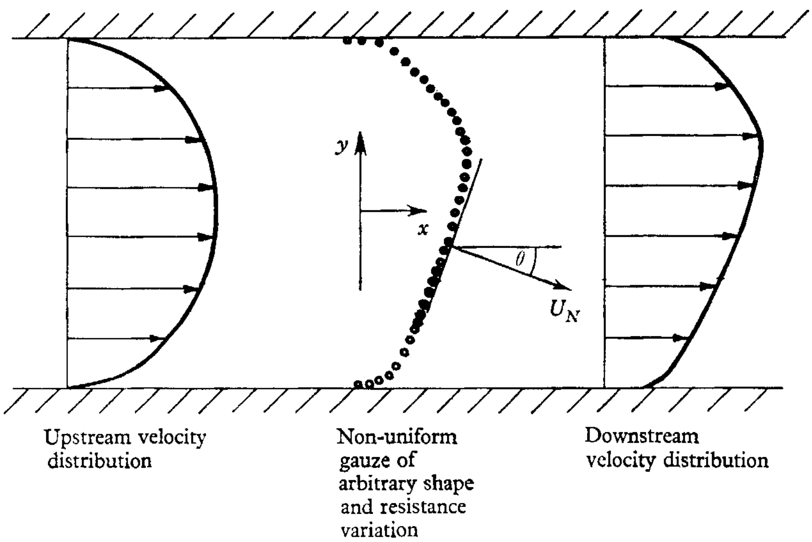

FIGURE 1. The general problem of two-dimensional flow through a non-uniform gauze.

The equation relating the velocity changes to the gauze properties may be written as

$$u - u^* = \gamma_0(q-1) + \tfrac{1}{2}\gamma_0 s. \tag{3}$$

In this expression, $u = u_{\infty 1}/\overline{U}$ and $u^* = u_{\infty 2}/\overline{U}$ are the non-dimensional velocities, distant from the plane of the gauze, in the upstream and downstream flow respectively. At the gauze surface, $q = u_1/\overline{U}$ is the local flow velocity and the effective loss coefficient γ, which depends not only on the loss coefficient K but also on the angle between the gauze normal and the flow direction θ, is written as

$$\gamma = K\cos^2\theta. \tag{4a}$$

On the assumption that the angle θ and variation in resistance across the gauze are both small, the right-hand side of (4a) may be linearized to the form

$$\gamma = \gamma_0(1+s). \tag{4b}$$

Continuity requires that $\int_0^L s\,dy = 0$. To satisfy the assumptions in (4), it will be necessary to restrict the velocity shear and the inclination of the flow to the gauze. Thus, $L(du/dy)$ or $L(du^*/dy) < 0{\cdot}5$ and $\theta < 45$ approximately.

Considering that the gauze produces a perturbation ψ^* to the main flow stream function ψ_0, then the resultant stream function is

$$\psi = \psi_0 + \psi^*.$$

Assuming small transverse streamline displacement, the perturbation satisfies the Laplace equation

$$\nabla^2\psi^* = 0,$$

which has a solution

$$\left.\begin{aligned}\frac{\psi^*}{LU} &= \sum_{n=1}^{\infty}\frac{P_n}{n\pi}\exp\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right) \quad \text{for} \quad x<0\\ &= \sum_{n=1}^{\infty}\frac{Q_n}{n\pi}\exp\left(-\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right) \quad \text{for} \quad x>0.\end{aligned}\right\} \tag{5}$$

In (5), P_n and Q_n are Fourier constants, x and y are the orthogonal co-ordinates and L is the channel height.

Assuming that the gauze is nearly coincident with the plane $x = 0$, the velocities given by the stream function ψ^* must satisfy continuity, so that

$$q = u - \sum_{n=1}^{\infty} P_n \cos n\omega = u^* - \sum_{n=1}^{\infty} Q_n \cos n\omega, \tag{6}$$

where $\omega = \pi y/L$ is a non-dimensional height parameter.

Hence the problem reduces after some manipulation of the algebra to the solution of the equations

$$u - u^* = \gamma_0(q-1) + \tfrac{1}{2}\gamma_0 s, \tag{7}$$

$$\tfrac{1}{2}\gamma_0 s + \gamma_0(u-1) = \sum_{n=1}^{\infty}\beta_n \sin n\omega, \tag{8}$$

$$B\tan\theta = \sum_{n=1}^{\infty}\alpha_n \sin n\omega, \tag{9}$$

in which P_n, Q_n are replaced by

$$\left.\begin{aligned}\alpha_n &= (1-B)P_n + Q_n,\\ \beta_n &= (1+\gamma_0)P_n - Q_n.\end{aligned}\right\} \tag{10}$$

Substitution for q from (6) and elimination of the constants P_n, Q_n reduces (7) to

$$u^* - 1 = A(u-1) - \tfrac{1}{2}(1-A)s + E\sum_{n=1}^{\infty}\alpha_n \cos n\omega, \tag{11}$$

where

$$E = \frac{\gamma_0}{2+\gamma_0-B} \quad \text{and} \quad A = \frac{2-\gamma_0-B+\gamma_0 B}{2+\gamma_0-B} = 1-\gamma_0(1-E).$$

In this equation, the final term presents a difficulty since the constants α_n are not easily evaluated. Elder was able to overcome the problem by introducing a type of transformation. If two functions $g(\omega)$, $g^*(\omega)$ are defined in the range $0 \leqslant \omega \leqslant \pi$ and are such that

$$g(\omega) = \sum_{n=1}^{\infty}\alpha_n \sin n\omega,$$

$$g^*(\omega) = \sum_{n=1}^{\infty}\alpha_n \cos n\omega,$$

then there is a transformation which satisfies

$$g(\omega) = H^*[g^*(\omega)],$$
$$g^*(\omega) = H[g(\omega)].$$

This transformation is a statement of a particular relationship between a Fourier sine series and its conjugate cosine series (Jeffreys & Jeffreys 1950, p. 431, etc.).

Hence, (9) and (11) yield

$$u^* - 1 = A(u-1) - \tfrac{1}{2}(1-A)\,s + E\,.\,H[B\tan\theta]. \tag{12}$$

This is a linear relationship between the flow and the gauze properties. The first term on the right-hand side represents the attenuation of an upstream flow variation by a uniform (plane, constant property) gauze normal to the flow, the second term describes the effect of resistance coefficient variation and the final term depends on the inclination of the gauze. Rearranging the equation, and using the transformation if necessary, gives any one variable in terms of the other four.

Two cases will be of practical interest: the shaped gauze of uniform resistance, and the plane normal gauze of non-uniform resistance. Either case may be related to a specified change in velocity distribution across the gauze. In the former problem, (12) is written as

$$B\tan\theta = H^*[\{(u^*-u)/E\} + (2-B)(u-1) + \tfrac{1}{2}(2-B)\,s], \tag{13}$$

which after insertion of the various terms into the bracketed section to form

$$g^*(\omega) = \sum_{n=1}^{\infty} \alpha_n \cos n\omega$$

and application of the transformation to produce

$$g(\omega) = \sum_{n=1}^{\infty} \alpha_n \sin n\omega$$

yields the gauze inclination θ as a function of y. Finally, the gauze shape is determined as

$$x - x_0 = \int_0^y \tan\theta\, dy. \tag{14}$$

Alternatively, where a plane gauze is considered, (12) reduces to

$$s = \left(\frac{2}{1-A}\right)[A(u-1) - (u^*-1)], \tag{15}$$

which expresses the required resistance variation in terms of the specified change in velocity distribution across the gauze.

Equations (12), (13) completely describe two-dimensional channel flow through a non-uniform gauze. Elder was able to obtain analytical solutions to certain well-chosen problems in which one or other of the terms vanished. The purpose of this paper is to show that in fact these equations may be solved quite generally using simple numerical techniques. Thus most experimental situations become amenable to analysis.

Computation procedure and experimental evaluation

Numerical solutions have been obtained using the Manchester University Atlas Computer. The methods adopted will be discussed briefly, making comparison with experiment where this is possible.

It is convenient to consider two distinct types of problem. Thus, it will be required to determine either (i) the velocity distribution downstream of a non-uniform gauze with a specified upstream velocity distribution, see (12), or (ii) the shape of the uniform gauze, or the resistance grading of a wire grid, which will produce a desired downstream velocity distribution from a given upstream distribution, see (13), (15).

Type (i). *The effect of a non-uniform gauze on a velocity distribution*

The basic steps in the calculation of the downstream velocity distribution u^* for a given distribution u and variation in gauze properties (inclination θ, loss coefficient K and lift coefficient B) are briefly the following: (a) compute the variation of gauze properties, using ($1a$), ($1b$), ($2a$), ($2b$) where necessary, and the geometry of the gauze; (b) specify the upstream velocity variation (u) at discrete values of the height parameter (ω); (c) determine the gauze constants γ_0, $s(\omega)$, A, E; (d) express the function $B \tan\theta$ as a Fourier series and determine the Fourier

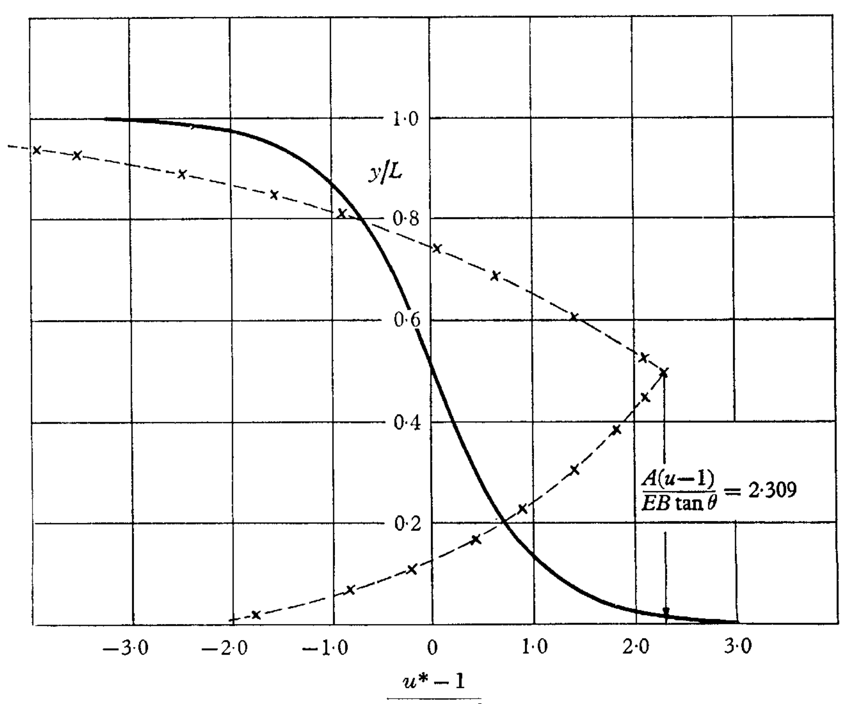

FIGURE 2. The influence of a plane inclined gauze on a uniform and a quarter power law velocity distribution. – – – × – – –, symmetrical quarter power velocity distribution upstream, $K = 1{\cdot}00$, $\theta = 45°$.

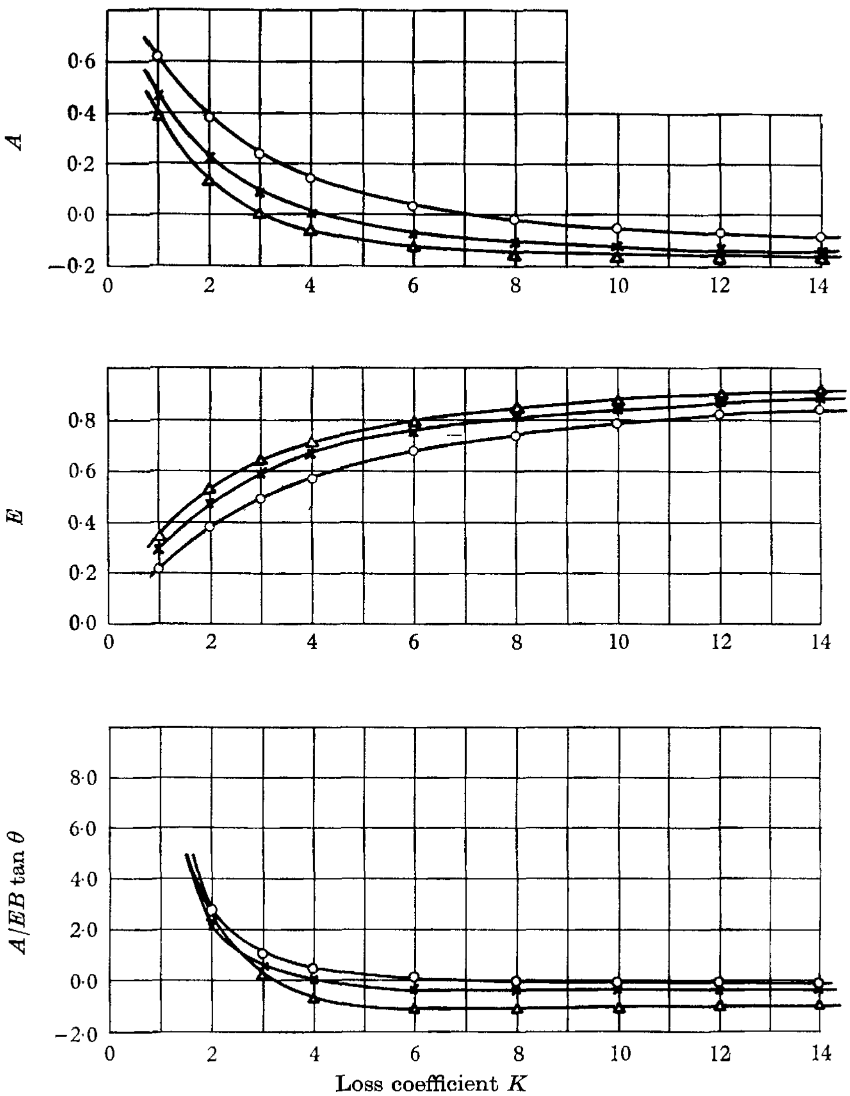

FIGURE 3. Gauze parameters associated with the influence of a plane inclined gauze on a non-uniform velocity distribution. △, 15°; ×, 30°; ○, 45°.

coefficients α_n. Hence evaluate the transformation using these coefficients; (e) compute the value of the right-hand side of (12) at each value of ω. This corresponds to the required downstream velocity distribution.

Two problems have been investigated in this way and will now be discussed.

A plane gauze inclined to the upstream flow. Computer solutions have been obtained for the velocity distribution downstream of a plane inclined gauze with both uniform and power law velocity distribution upstream, see figure 2. The computed velocity distributions for a uniform upstream flow were identical with the analytical results of Elder. Hence, agreement is found with Elder's experimental values, obtained with a negligibly small wall boundary-layer thickness. Using (12), the effect of non-uniformities in the upstream velocity distribution on the downstream distribution can be computed with relative ease. Figure 3, which shows the variation of the plane gauze constants $A, E, A/EB \tan\theta$ *versus* the loss coefficient K, may be used in conjunction with the information in figure 2. If the experimental upstream distribution $(u-1)$ is expressed as a discrete function of ω, the correction to the downstream distribution (u^*-1) can be calculated.

Parabolic gauze shape. The effect of a parabolic shaped gauze with a constant loss coefficient was analyzed by Elder. Unfortunately two printing errors appear in the original paper (pp. 366 and 367) which lead to some difficulty.

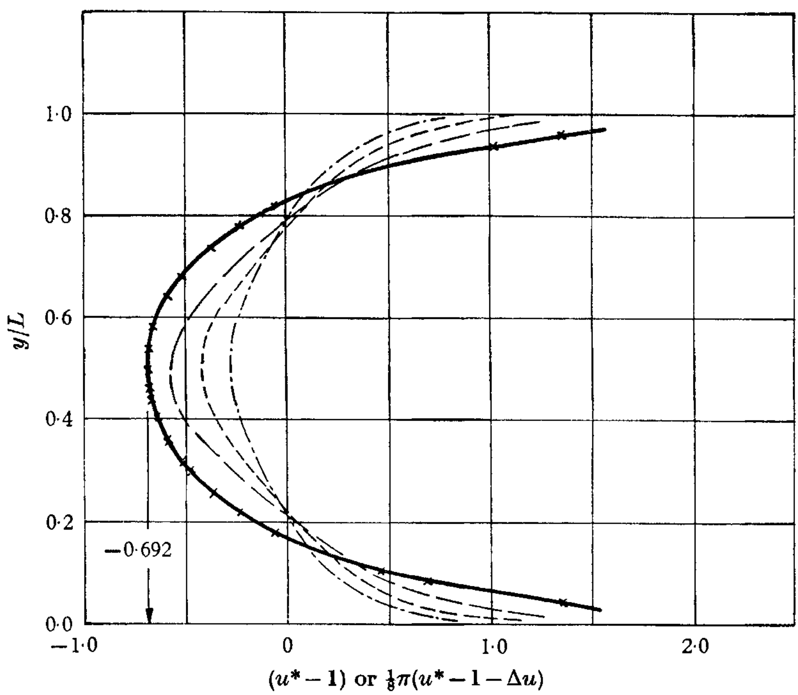

FIGURE 4. The effect of a parabolic gauze on a uniform velocity distribution. Gauze shape $(y-L/2)^2 = (L/4k)(kL-x)$. Curves of (u^*-1): ———, $k = 0{\cdot}30, K = 10{\cdot}0$; - - - - - - - -, $k = 0{\cdot}26, K = 4{\cdot}0$; - · - · - ·, $k = 0{\cdot}37, K = 4{\cdot}0$. ——×——, $\frac{1}{8}\pi(u^*-1-\Delta u)$.

Considering the gauze of shape

$$(y-\tfrac{1}{2}L)^2 = L(kL-x)/4k,$$

then the gauze shape correction term should be modified to

$$\Delta u = \tfrac{1}{2}(1-A)(1-4k\cos^2\theta/\tan^{-1}4k)$$

in the analytical expression for the downstream velocity distribution

$$(u^*-1)-\Delta u = \frac{-8}{\pi}kEB\log(2\sin\omega).$$

Secondly, the axis in Elder's figure 5 must be re-labelled $\frac{1}{8}\pi(u^*-1-\Delta u)/kEB$.

With these two corrections, the numerical solution was identical with the analytical solution already obtained by Elder and therefore agrees with the experimental results, see figure 4. Similar corrections for non-uniformity in the upstream velocity distribution to those discussed previously could be made. The determination of the gauze constants A and E is rather more difficult in this case where the gauze inclination changes with position but may be accomplished by numerical integration.

Type (ii). *The required gauze shape for a given change in velocity distribution*

In this type of problem, it is necessary to repeat the whole calculation of the gauze shape several times using an iterative procedure, since the variable s and therefore γ_0, A, E are functions of the inclination. The iterative procedure is basically as follows: (a) assume that $\gamma_0 = K$ so that initial approximations to the gauze parameters A and E may be computed. At this stage, the resistance variation term $s(\omega)$ in (13) is taken as zero; (b) choose a suitable interval for ω, at least 50 steps in the present work, and evaluate the bracketed term on the right-hand side of (13). This function of the velocity distributions and the resistance variation is denoted by $G_1(\omega)$; (c) to produce the transformation $H^*[g^*(\omega)]$, the function $G_1(\omega)$ is expressed as a Fourier cosine series, thus $G_1(\omega) = \sum_{n=1}^{N} \alpha_n \cos n\omega$ and the coefficients α_n are evaluated. The required transformation is simply the Fourier sine series $G_2(\omega) = \sum_{n=1}^{N} \alpha_n \sin n\omega$ obtained using the same coefficients α_n. The number of terms needed (N) has generally been found to lie between 15 and 30; (d) the gauze inclination θ may now be calculated as a function of height, using $\tan\theta = G_2(\omega)/B$; ($e$) integrating $\cos^2\theta$ across the gauze height permits the calculation of a more accurate value for the gauze constant γ_0, and of a value for the resistance variation $s(\omega)$. The complete calculation, starting at (b), is now repeated until there is no significant change in γ_0 or $s(\omega)$. Generally the number of iterations has been found to lie between 10 and 30; (f) the required gauze shape is computed by the integration of $\tan\theta = dx/dy$ across the duct height.

This particular method, for which the whole calculation must be repeated several times, has been used to compute the gauze shape necessary for the production of a linear shear velocity distribution. The second example quoted, that in which the required downstream distribution is produced by a grid formed by an array of parallel rods, illustrates a rather different method of solution in which the inclination θ is zero and some simplification of the problem occurs.

Production of a linear shear velocity distribution using a shaped gauze. Consider the linear shear velocity distribution

$$u^* - 1 = \lambda\{(y/L) - \tfrac{1}{2}\},$$

which is to be produced by a shaped gauze with constant properties. The problem may be tackled numerically: figure 5 shows the computed gauze shapes corresponding to a range of the velocity shear parameter λ and loss coefficient K. The effect of choosing equation ($2b$) instead of ($2a$) when calculating the lift coefficient B, is shown for one particular case $K = 6{\cdot}0$, $\lambda = 0{\cdot}8$, otherwise equation ($2a$) is used throughout. As mentioned earlier, errors in the lift coefficient are especially important in this type of problem since the gauze shape is determined by the rapidly varying function $B \tan \theta$.

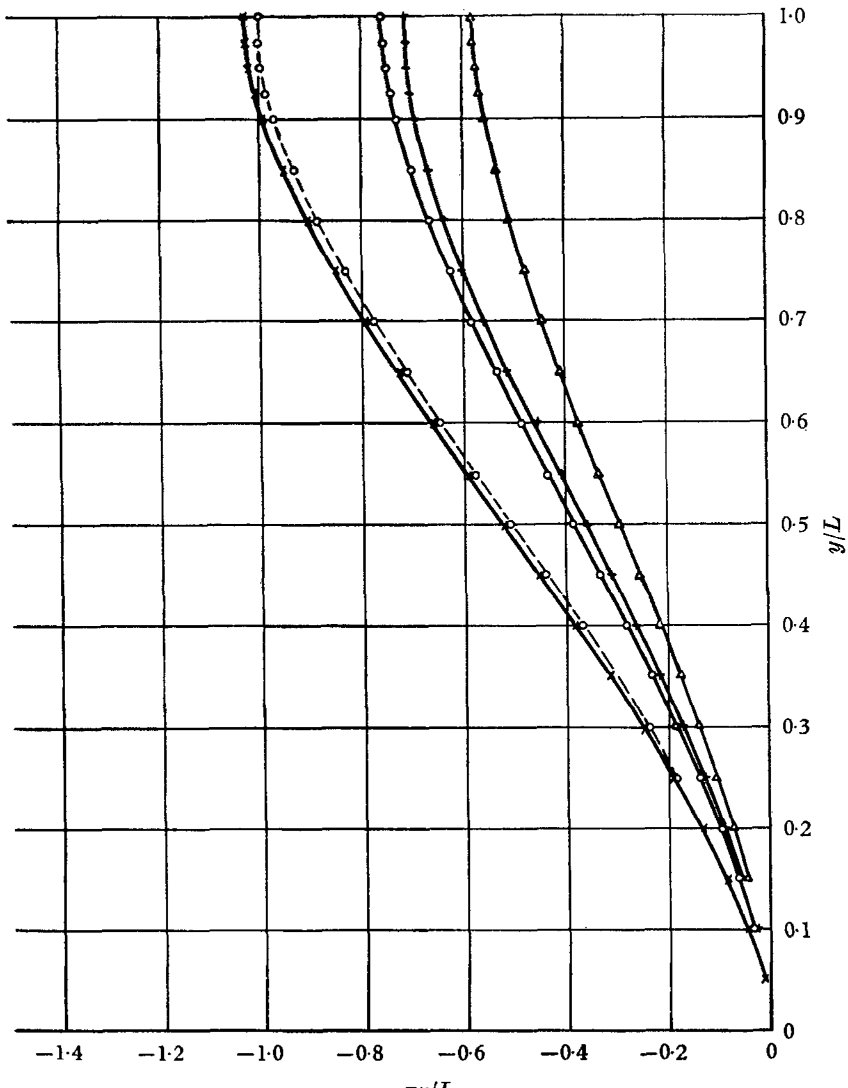

FIGURE 5. The uniform gauze shape to produce a linear shear velocity distribution: $u^* - 1 = \lambda(y/L - \frac{1}{2})$. Lift coefficient: ———, $B = 1 - 1/\surd(1 + \surd K)$. ×, $K = 4 \cdot 0$, $\lambda = 0 \cdot 8$; ○, $K = 6 \cdot 0$, $\lambda = 0 \cdot 8$; +, $K = 2 \cdot 5$, $\lambda = 0 \cdot 35$; △, $K = 4 \cdot 2$, $\lambda = 0 \cdot 5$ (see Lau & Baines (1968)). – – ○ – –, $B = 1 - 1 \cdot 1/\surd(1 + K)$.

It will be obvious that the computed shapes are very different from those derived analytically by Elder, particularly close to the walls where the plane inclined gauze requires the most correction. It is found that the computed gauze shape is practically unchanged if the resistance variation term $s(\omega)$ is neglected, as in Elder's analysis, so that an error in the evaluation of the transformation $H^*[g^*(\omega)]$ is indicated. In regard to this discrepancy, it has come to the author's attention that Lau & Baines (1968) have independently obtained agreement with the numerical work presented here. They have also discussed the reason for the error in the analytical solution due to Elder.

In order to test the numerical solution, a shaped gauze ($K = 2{\cdot}5$, $\lambda = 0{\cdot}35$) was installed in a wind tunnel with a 5 in. square cross-section so as to produce a vertical linear shear velocity distribution. Conveniently, the background turbulence level in this tunnel is relatively low. Considerable difficulty was experienced in the manufacture of this shaped gauze—eventually, the problem was overcome by soldering the gauze to vertical formers attached to the side walls and fixing the top and bottom edges of the gauze in slots machined in the horizontal walls of the tunnel. Only a small amount of side tension was applied.

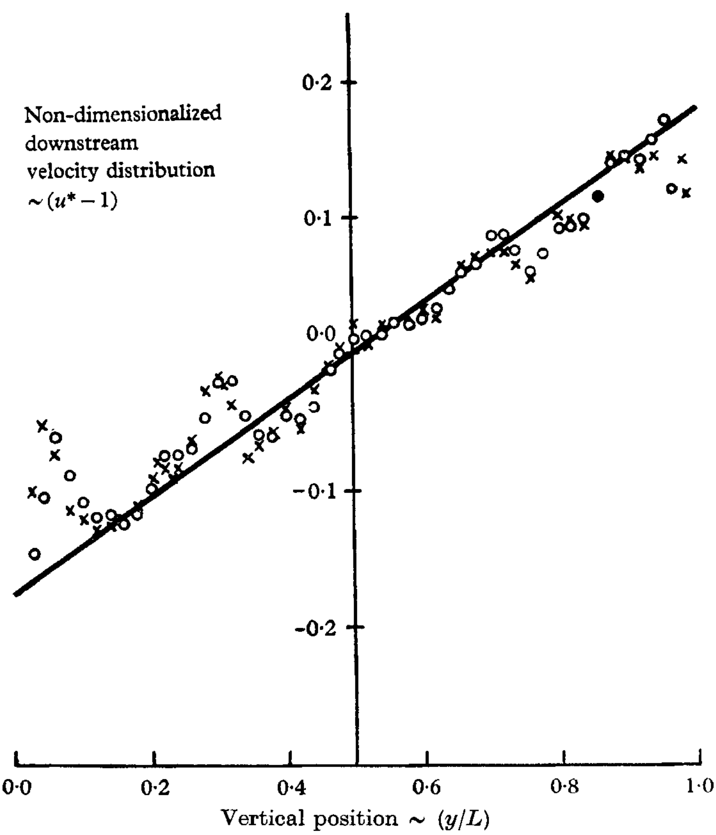

FIGURE 6. The production of a linear shear velocity distribution using a shaped gauze with uniform properties. ———, $u^* - 1 = 0{\cdot}35\{(y/L) - \frac{1}{2}\}$. Measured value: ×, at $3\frac{1}{2}$ in.; ○, at $7\frac{1}{4}$ in.

Measurements of the local total pressure and wall static pressure were obtained at $3\frac{1}{2}$ in. and $7\frac{1}{4}$ in. downstream from the leading edge of the gauze (corresponding to the minimum velocity in the downstream flow). These results are compared with the theoretical distribution in figure 6. Despite the scatter (5 % approx.), the data is essentially linear. Departures from the theoretical distribution may be attributed to (*a*) small unavoidable ripples in the gauze shape which produced the discrepancies to be observed at $y/L = 0{\cdot}26$ and $0{\cdot}7$, typically the ripple size was less than 0·002 inch, and (*b*) the wall boundary layers which modify (13) close to the walls.

Corrections for the non-uniformity in the flow upstream of the gauze could be computed if required although this has not been attempted in the present work. The corrections are basically the same as those considered for the plane inclined gauze.

The spacing distribution for a wire grid. One of the standard methods of velocity profile generation uses a grid of parallel rods with variable spacing. In the original work, Owen & Zienkiewicz (1957) derived the theoretical spacing to produce a linear shear velocity distribution. Subsequently, Cockrell & Lee (1966) extended the method to produce a power law distribution downstream of the grid. In both cases, the upstream flow was necessarily assumed to be uniform.

Using numerical techniques, the more general problem, in which the upstream velocity varies over the duct, can be investigated. The required spacing distribution may be obtained readily as the solution of a quadratic function.

The required resistance grading is

$$s = \{2/(1-A)\}\,[A(u-1)-(u^*-1)]. \tag{15}$$

Following Owen & Zienkiewicz, the resistance coefficient K is linearized and A can be determined.

Thus

$$K = K_0(1+s) \quad \text{and} \quad A = \frac{2-K_0-B+K_0B}{2+K_0-B}.$$

The drag coefficient for each rod based on interstitial velocity is assumed to equal 1·0 (use of McCarthy's value of 0·78 modifies the subsequent analysis only slightly) and hence it can be shown that

$$K_0(1+s) = \frac{\xi}{(1-\xi)^2} \quad \text{where} \quad \xi = \frac{d}{l}. \tag{16}$$

Substitution in (15) yields

$$\left[\frac{\xi}{K_0(1-\xi)^2}-1\right] = \left(\frac{2}{1-A}\right)[A(u-1)-(u^*-1)]. \tag{17}$$

Solution of this quadratic then gives the required variation in wire spacing.

Thus

$$\xi = \frac{(2X+1)-\sqrt{(4X+1)}}{2X}, \tag{18}$$

where we define

$$X = K_0\left\{1+\left(\frac{2}{1-A}\right)[A(u-1)-(u^*-1)]\right\}. \tag{18a}$$

The negative square root is taken since the distance between adjacent rods is finite ($0 \leqslant \xi \leqslant 1$).

In fact, the equation solved by Cockrell & Lee (1966) differs from (17) due to the choice of the resistance coefficient expression. Following Elder they used

$$K_0(1+s) = \frac{\xi^2}{(1-\xi)^2}. \tag{19}$$

Substitution into (15) and solution by the quadratic formula leads to the wire spacing distribution

$$\xi = \frac{-(K_0+Y)+\surd(K_0+Y)}{1-(K_0+Y)}, \tag{20}$$

where

$$Y = \left(\frac{2K_0}{1-A}\right)[A(u-1)-(u^*-1)]. \tag{20a}$$

For this particular case, only the positive square root is applicable since physically $0 \leqslant \xi \leqslant 1$ for the rod array.

In addition to the different expressions for the loss coefficient variation, there is some uncertainty in the choice of the correct expression for the lift coefficient. Thus Owen & Zienkiewicz originally used expression (2*b*) whereas Cockrell & Lee adopted the relationship derived by Elder, (2*a*).

It is noteworthy that, for the special case of a square mesh gauze, (2*a*) is equivalent to the expression

$$B = 1 - 1/\surd(1+\surd K), \tag{2c}$$

since $\beta = \{1-(d/l)\}^2$, provided that we assume

$$K = \left(\frac{1-\beta}{\beta}\right)^2. \tag{1a}$$

For an array of parallel cylinders, $\beta = \{1-(d/l)\}$ and equations (1*a*), (2*a*), (2*c*) are not compatible.

The variations in the value of B, corresponding to the different relationships available, lead to changes in the constant A. The resultant changes in the spacing distribution are however found to be relatively small. Typically, the changes are less than 2 % in ξ for $y/L < 0{\cdot}8$ and less than 5 % for $y/L > 0{\cdot}8$. Changes of this order are practically insignificant.

In contrast to these small variations, the alternative expressions for the effective resistance coefficient of the grid ((16) or (19)), lead to very different results for the calculated grid spacing distribution. Figure 7 illustrates this for the cases of the production of a uniform shear and a one-seventh power law velocity distribution, starting with a constant upstream velocity.

Since only one expression can be correct, the good agreement obtained by different workers between theory and experiment is surprising. However, it is found that changes in the value of the assumed loss coefficient will reduce (but not eliminate) the disparity between the alternative spacing distributions; see figure 7 in the case of the one-seventh power law. It seems probable that the assumed (i.e. theoretical) and the experimental K_0 values were not necessarily of equal value. It is suggested that this discrepancy could have been overlooked previously since the downstream velocity distribution, rather than the pressure loss across the grid, would have the primary importance.

A careful examination of the experimental power law velocity distributions produced by Cockrell & Lee (1966) shows that their 0·143 ($\frac{1}{7}$) distribution is fitted rather more closely by a 0·175 (1/5·7) law. Employing this modified power law index and choosing a value of 1·00 (instead of the 0·4 assumed), it is found that (16) leads to spacing distributions which are practically identical with those

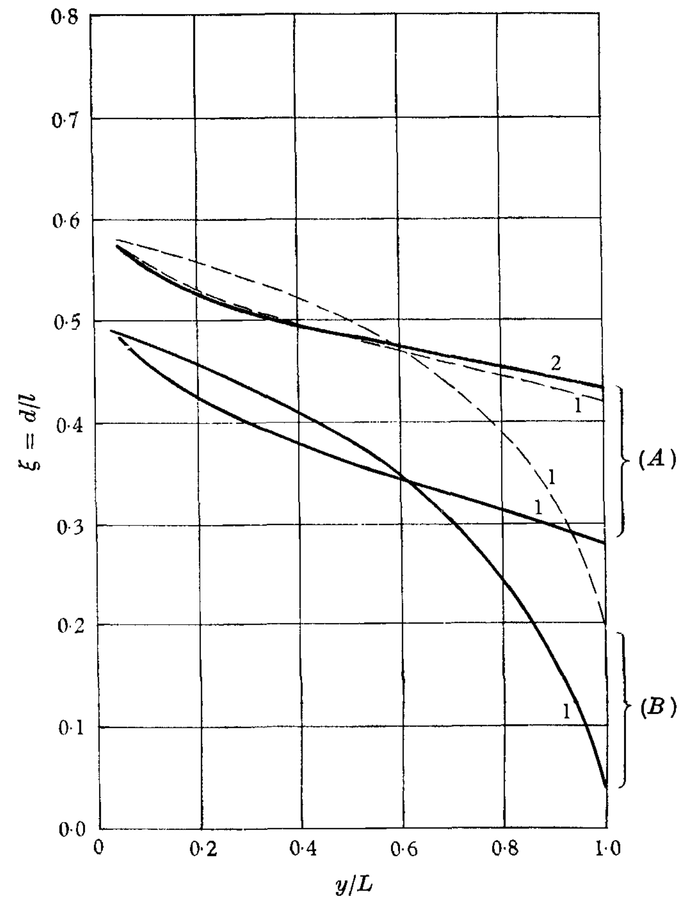

FIGURE 7. The influence of the alternative expressions for the resistance coefficient on the computed spacing distribution of a grid. ———, $K_0(1+s) = \xi/(1-\xi)^2$, $B = 1-1\cdot1/\sqrt{(1+K_0)}$; - - - - -, $K_0(1+s) = \xi^2/(1-\xi)^2$, $B = \xi$. Curves numbered: 1, $K_0 = 1\cdot0$; 2, $K_0 = 2\cdot0$. (*A*), power law, $u^* = \frac{8}{7}(y/L)^{\frac{1}{7}}$; (*B*), linear shear, $u^*-1 = 0\cdot6\{(\frac{1}{2}-y/L)\}$.

given by Cockrell & Lee—curves 4 and 2 in figure 8. In fact, a straight line approximation to the spacing distribution was used in these experiments so that the actual spacing distribution was perhaps even nearer to curve 4 than the ξ values calculated by Cockrell & Lee would suggest. With the modified value of the power law index (5·7 not 7·0) such close agreement could not be obtained using the alternative equation (19), curves 1 and 3 in figure 8 with K_0 values of 0·45 and 0·50.

The choice between the two expressions (16) or (19) need not be entirely arbitrary. The available evidence based on the work of Owen & Zienkiewicz (1957), McCarthy (1964) and on this present re-analysis of the results of Cockrell & Lee (1966) suggests that (16), possibly modified by the constant factor 0·78, is the more correct.

It is therefore suggested that (18) should be employed for grid design. Since

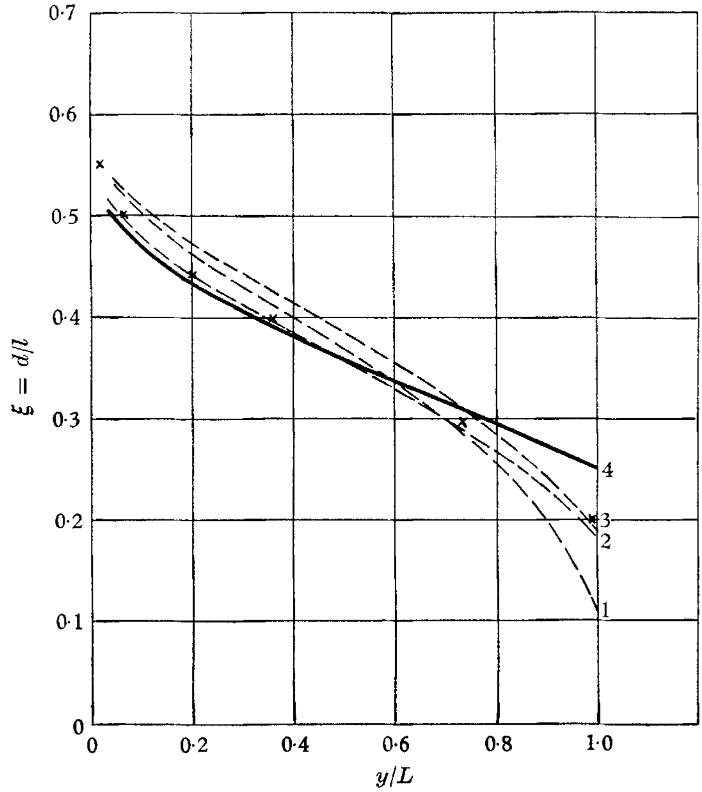

FIGURE 8. A re-analysis of the grid spacing distribution tested by Cockrell & Lee (1966). Power law: $u^* = \{(n+1)/n\}\,(y/L)^{1/n}$. ———, $K_0(1+s) = \xi/(1-\xi)^2$, $B = 1-1{\cdot}1/\sqrt{(1+K_0)}$; – – – – – –, $K_0(1+s) = \xi^2/(1-\xi)^2$, $B = \xi$. ×, values calculated by Cockrell & Lee, $n = 7$, $K_0 = 0{\cdot}4$. Curve no.: 1, $n = 5{\cdot}7$, $K_0 = 0{\cdot}45$; 2, $n = 7{\cdot}0$, $K_0 = 0{\cdot}40$; 3, $n = 5{\cdot}7$, $K_0 = 0{\cdot}50$; 4, $n = 5{\cdot}7$, $K_0 = 1{\cdot}00$.

the value of the constant A is small for practicable values of the loss coefficient K_0, the effect of any non-uniformity in the upstream flow will usually be small.

The numerical method of solution is seen to be easily adapted to any particular experimental problem. The spacing of an array of parallel rods (i.e. the wire grid) has been analyzed in rather more detail than the other types of gauze problem because, in two-dimensional flows at least, this method is more convenient experimentally than the bending of an accurate gauze shape. However, for other situations, and particularly in axisymmetric flows, the use of shaped uniform gauzes appears to offer certain advantages. For this reason the considerable complications of the axisymmetric problem are currently being investigated.

Conclusion

A computational method has been presented for the analysis of the two-dimensional flow through a non-uniform gauze of arbitrary shape. The method is applicable to the most general, and therefore analytically difficult, practical cases.

The various types of problem may be solved using relatively simple numerical techniques which are easily programmed onto a digital computer and good agreement has been obtained between experiment and the computed results. In particular, the computed shape of the uniform gauze needed to produce a linear downstream velocity distribution has been tested by experiment since, in this one case, there is disagreement with the results presented by Elder. The solution obtained numerically is shown to be correct although the experimental results are of rather poor quality.

The author wishes to express his sincere thanks to Dr P. H. Price for help with the analysis and Mr S. Bates and Mr J. Toft for constructing the experimental apparatus. He is also very grateful to Dr D. J. Cockrell and Mr B. E. Lee for making their experimental results available for further analysis.

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